McPeak
Lecture 3
PAI 723

Now we move to the topic of quantitative change. Not only the question of which direction, but how much in that direction.

We might be interested in the "how much" question. For planning purposes, we may be interested not only in what direction, but how much in a given direction we will move as a result of a given policy.

For this, we will see that the shape of the curves matter. How steep are our curves?

Let us consider a shift in the demand curve. Where the new equilibrium lies will depend on the slope of the supply curve.

Say for example that the price of beef goes up and we are still considering our processed pork example.

We could do it by an equation by equation approach, or a graph by graph approach.

Review the impact of the price of beef going from 4 to 5.
Q=171-20*p+(20*4)+(3*3.33)+(2*12.5)
Q=286-20*p
Compared to:
Q=171-20*p+(20*5)+(3*3.33)+(2*12.5)
Q=306-20*p

If the price of beef goes from 4 to five, and chicken price and income is constant,

Price	Quantity if p_b=4	Quantity if p_b=5
5	286-20*5 = 186	306-20*5 = 206
4	286-20*4 = 206	306-20*4 = 226
3	286-20*3 = 226	306-20*3 = 246
2	286-20*2 = 246	306-20*2 = 266
1	286-20*1 = 266	306-20*1 = 286

Supply curves:

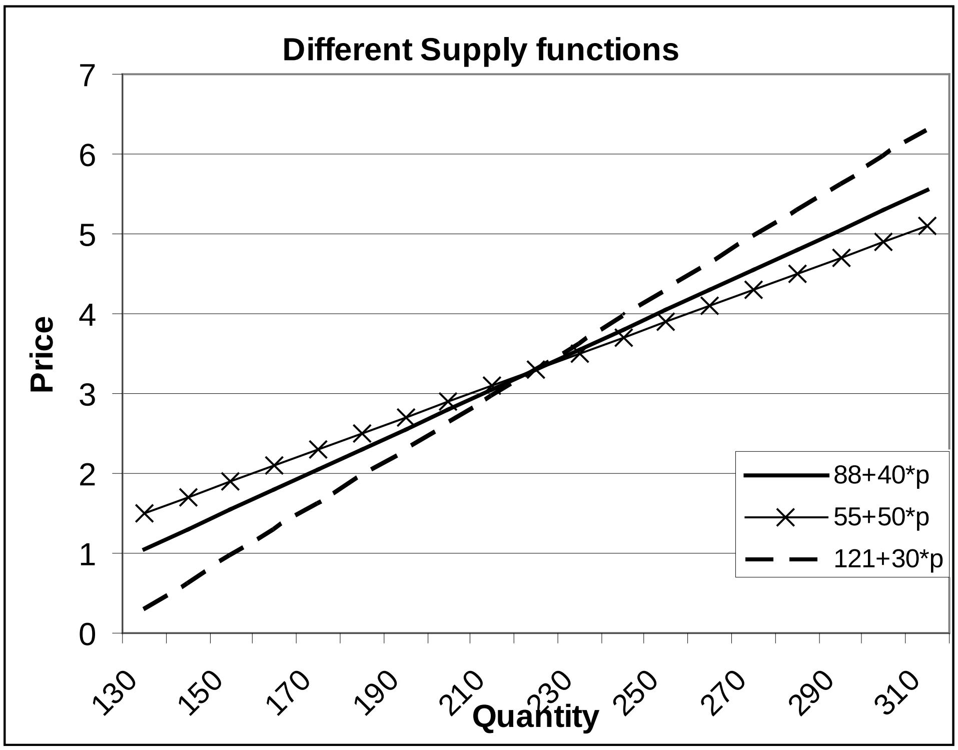
Consider a supply curve defined by Q^s=55+50*p or ($p = \frac{1}{50} \cdot Q - \frac{55}{50}$)

Originally given Q^s=88+40*p or ($p = \frac{1}{40} \cdot Q - \frac{88}{40}$)

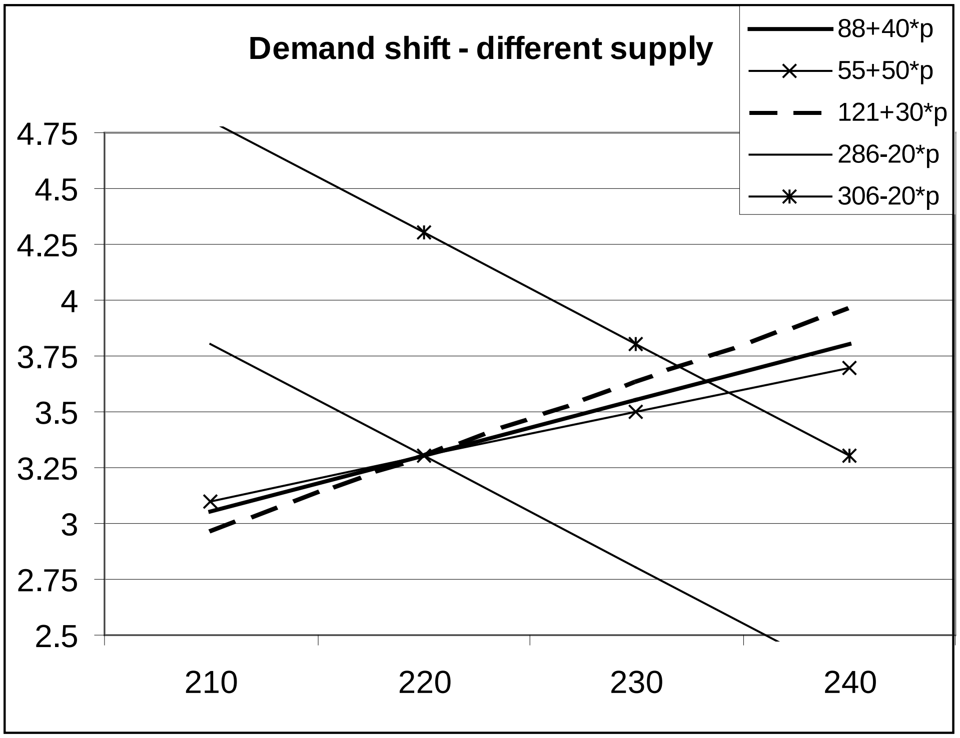
Consider a supply curve defined by Q^s=121+30*p or ($p = \frac{1}{30} \cdot Q - \frac{121}{30}$)

Price	Q^s=55+50*p “flat”	Q^s=88+40*p “original”	Q^s=121+30*p “steep”
5	55+50*5=305	88+40*5=288	121+30*5=271
4	55+50*4=255	88+40*4=248	121+30*4=241
3	55+50*3=205	88+40*3=208	121+30*3=211
2	55+50*2=155	88+40*2=168	121+30*2=171
1	55+50*1=105	88+40*1=128	121+30*1=151

[How did I get these supply curves? I wanted them to all pass through $3.30, 220, so I plugged these in and moved the slope from 40 to 50 and then to 30 and in each case solved for the intercept term. To graph them, I solved for p as a function of q – for example p=(1/40)*q-(88/40)]



Add in the issue of demand shift:



What are the different implications?
"flat"
306-20*p=55+50*p
p=$3.59, q=234

"original"
306-20*p=88+40*p
p=$3.63, q=233

"steep"
306-20*p=121+30*p
p=$3.70, q=232

These are made up numbers, but it does show that the underlying numbers that determine the shape of the curve matter for where you end up in equilibrium if things change.

In many cases, we can use a simple measure of sensitivity to capture important information about quantitative change.

This is elasticity, a unitless, summary measure of sensitivity.

Elasticity.

The percentage change in one variable as a response to a given percentage change in another variable.

$$\frac{\%\ change\ x}{\%\ change\ y}$$

Supply Elasticity:

$$\eta = \frac{\%\Delta Q_s}{\%\Delta p}$$

eta = % change in quantity supplied divided by the percent change in price.

The symbol for change is delta, Δ.

Alternatively, define it as $\dfrac{\frac{\Delta Q_s}{Q_s}}{\frac{\Delta p}{p}}$, or $\dfrac{\Delta Q}{\Delta p} \cdot \dfrac{p}{Q}$

Below one we call inelastic.
1 is unit elastic.
Above one is elastic.
Infinity extremes are perfectly elastic, zero is perfectly inelastic

Note:
1) ______elastic___|_____inelastic____|___elastic____
$-\infty$ -1 0 1 ∞
2) Intuition behind word “elastic”.
3) Calculus link.

“flat”
Try the calculation for Q_s =55+50*p, that led us to the p=$3.59, q=234 pair. Remember that we moved from an equilibrium pair of (3.30, 220).

Change in q: 220 to 234 = 14 units q
Change in p: 3.30 to 3.59 = $0.29

$\dfrac{\frac{\Delta Q_s}{Q_s}}{\frac{\Delta p}{p}}$	$\dfrac{\Delta Q}{\Delta p} \cdot \dfrac{p}{Q}$	$\eta = \dfrac{\%\Delta Q_s}{\%\Delta p}$
$\dfrac{\frac{14KG}{220KG}}{\frac{\$0.29}{\$3.30}}$	$\left(\dfrac{14KG}{\$0.29}\right) \cdot \left(\dfrac{\$3.30}{220KG}\right)$	$\%\Delta Q = (14KG / 220KG) = 6.36\%$ $\%\Delta p = (\$0.29 / \$3.30) = 8.79\%$

η=0.72

“original”
Consider the original curve. Remember that we moved from an equilibrium pair of (3.30, 220) to the equilibrium pair (3.63, 233) when we used Q_s =88+40*p.

Change in q: 220 to 233 = 13 units q
Change in p: 3.30 to 3.63 = 33 cents

$$\frac{\frac{\Delta Q_s}{Q_s}}{\frac{\Delta p}{p}}$$	$$\frac{\Delta Q}{\Delta p} \cdot \frac{p}{Q}$$	$$\eta = \frac{\%\Delta Q_s}{\%\Delta p}$$
$$\frac{\frac{13KG}{220KG}}{\frac{\$0.33}{\$3.30}}$$	$$\left(\frac{13KG}{\$0.33}\right) \cdot \left(\frac{\$3.30}{220KG}\right)$$	$\%\Delta Q = (13KG / 220KG) = 5.9\%$ $\%\Delta p = (\$0.33 / \$3.30) = 10\%$

η=0.59

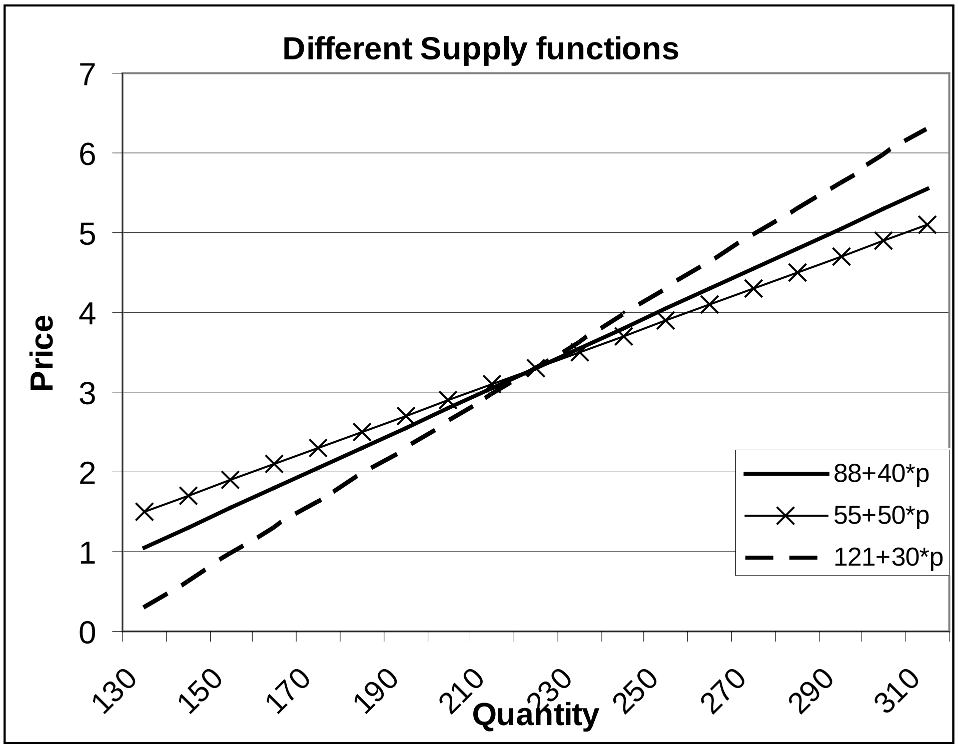
“steep”
How about the Q=121+30*p supply curve that took us to p=$3.70, q=232 when demand shifted?

Change in q: 220 to 232 = 12 units q
Change in p: 3.30 to 3.70 = 40 cents

$\dfrac{\dfrac{\Delta Q_s}{Q_s}}{\dfrac{\Delta p}{p}}$	$\dfrac{\Delta Q}{\Delta p} \cdot \dfrac{p}{Q}$	$\eta = \dfrac{\%\Delta Q_s}{\%\Delta p}$
$\dfrac{\dfrac{12KG}{220KG}}{\dfrac{\$0.40}{\$3.30}}$	$\left(\dfrac{12KG}{\$0.40}\right) \cdot \left(\dfrac{\$3.30}{220KG}\right)$	$\%\Delta Q = (12KG / 220KG) = 5.45\%$ $\%\Delta p = (\$0.40 / \$3.30) = 12.1\%$

η=0.45

Which was the most sensitive to a change in price? The one with the highest elasticity has the highest change in the quantity supplied for a given change in price. That is our 55+50*p supply curve.



This is a general pattern to keep in mind, but don't get too caught up in. A flatter curve as drawn above has more response in quantity for a given change in price than a steeper curve. Steep curves tend to be inelastic (price change does not bring about much change in quantity). Think of our steepest curve, the coefficient on the price variable is 30. For our flattest curve, it is 50. Get the idea?

In demand analysis, we are often interested in the price elasticity of the quantity demanded.

What is the percentage change in the quantity demanded, divided by the percentage change in the price?

Use the Greek letter epsilon , ε.
Recall that the symbol for change is delta, Δ.

$$\varepsilon = \frac{\%\Delta \mathrm{Q_d}}{\%\Delta \mathrm{p}} = \frac{\Delta \mathrm{Q} / \mathrm{Q}}{\Delta \mathrm{p} / \mathrm{p}}$$

Can state this in an equivalent fashion.

$$\varepsilon = \left(\frac{\Delta Q}{\Delta p} \right) \cdot \left(\frac{p}{Q} \right)$$

Now, in our demand shift case, we don’t have the information we need for the elasticity calculation from the previous example where we calculated a supply elasticity.

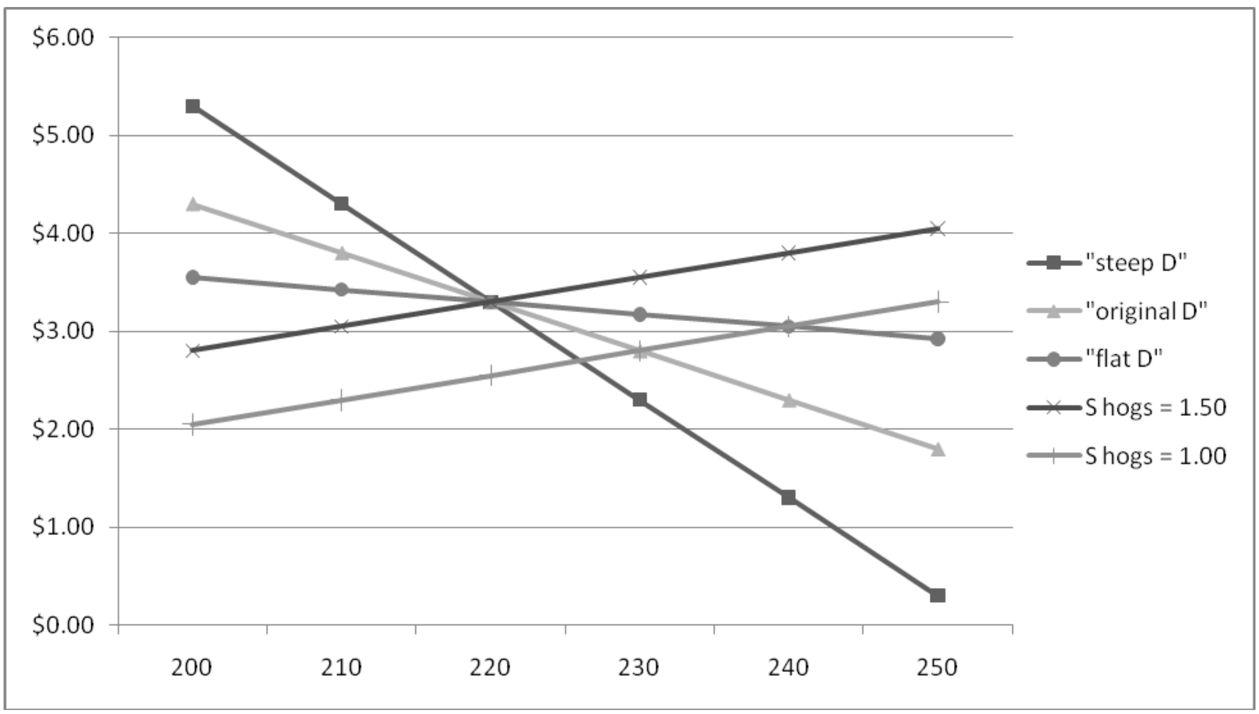
NOTE: THE PRICE ELASTICITY OF DEMAND IS ABOUT MOVEMENT ALONG A DEMAND CURVE, NOT A SHIFT IN A DEMAND CURVE. We had a demand shift giving us two points on the supply curve in each case above to work with. Now for the demand elasticity, I have to generate some kind of supply shift to get a similar story going.

So, go back to our original story yet again.

Q^d=286-20*p
Q^s=88+40*p

Remember, this took us to the equilibrium point p=$3.30, q=220.

Compare alternative demand curves, as I did before for the alternative supply curves. I will pick one steeper, one flatter.



If we have the given supply shift, we can see along the alternative demand curves. Recall that the processed pork supply curve was a function of the hog price. Assume the hog price decreases from $1.50 to $1.00. This leads to a downward shift in supply (I can produce more pork at a given selling price since my input cost decreased).

Q_s=118+40*p according to the information in the book following this price decrease for the input.

Again recall \$3.30, 220 is how we start.

Consider three alternative demand curves.

"flat"
The flattest case is Q_d=484-80*p / $p = 6.05 - .0125 * q$? Solve for the new equilibrium (484-80*p=118+40*p), find p=\$3.05, q=240.

Change in q is 20 (240-220).
Change in p is -\$0.25 (\$3.05-\$3.30)

$\dfrac{\frac{\Delta Q_D}{Q_D}}{\frac{\Delta p}{p}}$	$\dfrac{\Delta Q}{\Delta p} \cdot \dfrac{p}{Q}$	$\varepsilon = \dfrac{\% \Delta Q_D}{\% \Delta p}$
$\dfrac{\frac{20KG}{220KG}}{\frac{-\$0.25}{\$3.30}}$	$\left(\dfrac{20KG}{-\$0.25}\right) \cdot \left(\dfrac{\$3.30}{220KG}\right)$	$\% \Delta Q = (20KG / 220KG) = 9.1\%$ $\% \Delta p = (-\$0.25 / \$3.30) = -7.6\%$

ε =-1.2

The original case Q_d=286-20*p / $p = 14.3 - .05 * q$? Solve for the new equilibrium (286-20*p=118+40*p), find p=$2.80, q=230.

Change in q is 10. (230-220)
Change in p is -$0.50. ($2.80-$3.30)

$\dfrac{\dfrac{\Delta Q_D}{Q_D}}{\dfrac{\Delta p}{p}}$	$\dfrac{\Delta Q}{\Delta p} \cdot \dfrac{p}{Q}$	$\varepsilon = \dfrac{\%\Delta Q_D}{\%\Delta p}$
$\dfrac{\dfrac{10KG}{220KG}}{\dfrac{-\$0.50}{\$3.30}}$	$\left(\dfrac{10KG}{-\$0.50}\right) \cdot \left(\dfrac{\$3.30}{220KG}\right)$	$\%\Delta Q = (10KG / 220KG) = 4.5\%$ $\%\Delta p = (-\$0.50 / \$3.30) = -15.2\%$

ε =-0.3

“steep”
Solve for Q_d=253-10*p / $p = 25.3 - .1 * q$. If you solve this one for the new equilibrium after the shift (253-10*p=118+40*p), you get p=$2.70, q=226.

Change in q: 6. (226 -220)
Change in p: -$0.60. ($2.70-$3.30)

$\dfrac{\dfrac{\Delta Q_D}{Q_D}}{\dfrac{\Delta p}{p}}$	$\dfrac{\Delta Q}{\Delta p} \cdot \dfrac{p}{Q}$	$\varepsilon = \dfrac{\% \Delta Q_D}{\% \Delta p}$
$\dfrac{\dfrac{6KG}{220KG}}{\dfrac{-\$0.60}{\$3.30}}$	$\left(\dfrac{6KG}{-\$0.60}\right) \cdot \left(\dfrac{\$3.30}{220KG}\right)$	$\% \Delta Q = (6KG / 220KG) = 2.7\%$ $\% \Delta p = (-\$0.60 / \$3.30) = -18.2\%$

ε =-0.15

Recall:

______elastic____|______inelastic_____|____elastic_____
-∞ -1 0 1 ∞

For supply elasticities we found: 0.72, 0.59, 0.45.
For demand elasticities we found: -1.2, -0.3, -0.15

Place these on number line, contrast:
Supply elasticities tend to be on the positive side.
Demand elasticities are always on the negative side.
Inelastic and elastic areas.

Also can speak of elasticity in terms of absolute value: Less than one in absolute value is inelastic, greater than one is elastic.

Realize that a calculated elasticity may only be applicable in the neighborhood of the equilibrium, not for the entire demand curve.

Note that these calculations are for a given point on the curve. Take the example of the baseline curve, that had a constant slope of –(1/20). Q^d=286-20*p that is expressed as inverse demand of p=14.3-0.05*q.

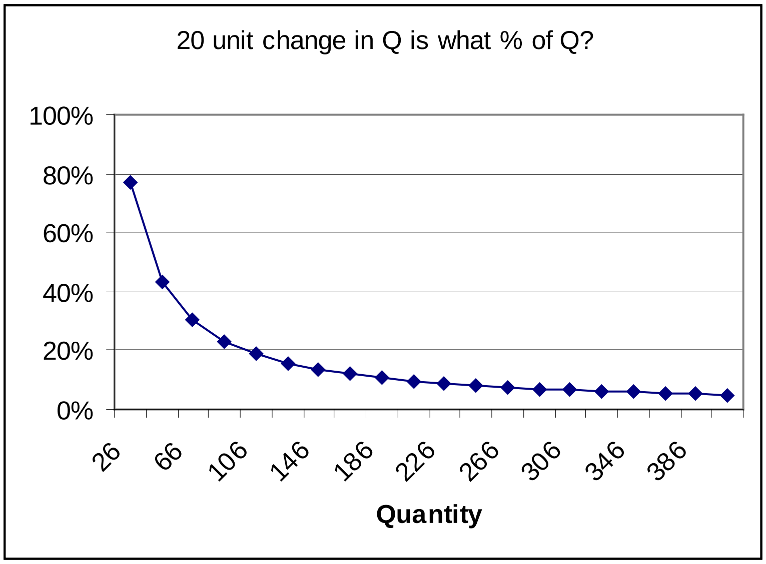
Price	Quantity if p_b=4
5	286-20*5 = 186
4	286-20*4 = 206
3	286-20*3 = 226
2	286-20*2 = 246
1	286-20*1 = 266

	Δp	ΔQ	p	Q	ε
1 to 2	1	-20	1	266	-0.08
2 to 3	1	-20	2	246	-0.16
3 to 4	1	-20	3	226	-0.27
4 to 5	1	-20	4	206	-0.39

Note relative ease of using this version of the formula.

$$\frac{\Delta Q}{\Delta p} \cdot \frac{p}{Q}$$

A constant slope is not the same as a constant elasticity.



Elasticity is a result relevant to the area around your equilibrium.

Also note the concept of arc elasticity, where you take the average of the starting and the ending points in defining p and q.

	Δp	ΔQ	Ave p	Ave Q	ε
1 to 2	1	-20	1.5	256	-0.12
2 to 3	1	-20	2.5	236	-0.21
3 to 4	1	-20	3.5	216	-0.32
4 to 5	1	-20	4.5	196	-0.46

Arc elasticity:

$$\frac{\Delta Q}{\Delta p} * \frac{.5*(p_1+p_2)}{.5*(q_1+q_2)}$$

Two other elasticities used in demand analysis: now we are looking at sensitivity of "shifts" in the curve, rather than sensitivity in terms of movement along in response to a shift.   A change in the all else equal set of variables.

Income Elasticity
What is the percentage change in the quantity demanded, divided by the percentage change in the income level that brings about this change in quantity demanded?

$$\xi = \frac{\Delta Q / Q}{\Delta Y / Y}$$

xi is the greek symbol used here.

Take the example of the processed pork again. Recall the baseline equation:

$$Q = 171 - 20 * p_p + 20 \cdot p_b + 3 * p_c + 2 \cdot Y$$

P_b=4, P_c=3.333, Y=12.5

Assume a one unit change in income from 12.5 to 13.5, so you can use the coefficient on the income variable in the quantity demanded equation.

Also, recall the baseline equilibrium result of: (p^*, q^*) = ($3.30, 220).

$$220 = 171 - 20 * 3.30 + 20 \cdot 4 + 3 * 3.33 + 2 \cdot 12.5$$

$$222 = 171 - 20 * 3.30 + 20 \cdot 4 + 3 * 3.33 + 2 \cdot 13.5$$

Change in q? 2.
Q? 220.
Change in Y? We assumed it to be1.
Y? 12.5.

$\dfrac{\frac{\Delta Q_D}{Q_D}}{\frac{\Delta Y}{Y}}$	$\dfrac{\Delta Q}{\Delta Y} \cdot \dfrac{Y}{Q}$	$\varepsilon = \dfrac{\% \Delta Q_D}{\% \Delta Y}$
$\dfrac{\frac{2KG}{220KG}}{\frac{\$1.00}{\$12.50}}$	$\left(\dfrac{2KG}{\$1.00}\right) \cdot \left(\dfrac{\$12.50}{220KG}\right)$	$\% \Delta Q = (2KG / 220KG) = .91\%$ $\% \Delta Y = (\$1.00 / \$12.50) = 8.0\%$

$$\xi = 25 / 220, or \quad 0.114$$

A normal good is one for which the income elasticity is positive.

An inferior good is one for which the income elasticity is negative.

Inferior goods tend to be things like staple foods. Not a “bad”, mind you, but something that you will consume less of as your income increases.

Brazil 1974-75. Income elasticity for cassava, -1.59, for rice, 0.172, for milk 0.147, for eggs, 0.630.

Shows the relationship between income and quantity demanded holding prices constant.

Example of economic models of the demand for children: are children a normal or inferior good?

Cross Price Elasticity
What is the percentage change in the quantity demanded, divided by the percentage change in the price of another good that brings about this change in quantity demanded?

$$\varepsilon = \frac{\Delta Q_1 / Q_1}{\Delta p_2 / p_2}$$

$$Q = 171 - 20 * p_p + 20 \cdot p_b + 3 * p_c + 2 \cdot Y$$

P_b=4, P_c=3.333, Y=12.5

Where there are two goods: good one and good two. Think of the pork example, and think a one unit change in the price of beef.

$$220 = 171 - 20 * 3.30 + 20 \cdot 4 + 3 * 3.333 + 2 \cdot 12.5$$

$$240 = 171 - 20 * 3.30 + 20 \cdot 5 + 3 * 3.333 + 2 \cdot 12.5$$

Change in Q? 20.
Q? 220.
Change in price of beef? We pick 1 dollar for ease of computation.
Price of beef? 4.

$\dfrac{\dfrac{\Delta Q_D}{Q_D}}{\dfrac{\Delta p}{p}}$	$\dfrac{\Delta Q}{\Delta p} \cdot \dfrac{p}{Q}$	$\varepsilon = \dfrac{\%\Delta Q_D}{\%\Delta p}$
$\dfrac{\dfrac{20KG}{220KG}}{\dfrac{\$1.00}{\$4.00}}$	$\left(\dfrac{20KG}{\$1.00}\right) \cdot \left(\dfrac{\$4.00}{220KG}\right)$	$\%\Delta Q = (20KG / 220KG) = 9.1\%$ $\%\Delta p = (\$1.00 / \$4.00) = 25\%$

Mix ingredients, and you get 0.36. If you want to practice, try the chicken price changing by one dollar and you should get 0.045.

A complement of a good is one for which the cross price elasticity is negative. (A 1% increase in the price of bacon leads to a -% change in the quantity demanded of eggs).

A substitute of a good is one for which the cross price elasticity is positive. (A 1% increase in the price of bacon leads to a + % change in the quantity demanded of sausage).

What have we found with the beef example here – a complement or a substitute for processed pork?

What does an elasticity mean?

Let’s go back to demand elasticities.

Goods that are relatively price inelastic mean that a large change in price leads to a relatively small change in the quantity demanded of the good.

Goods that are relatively price elastic mean that a small change in price leads to a relatively large change in the quantity demanded.

What determines the degree of elasticity?

1) Closeness of substitutes.
2) Time period over which these substitutes can be obtained.

Long run versus short run elasticities.

Goods tend to be more price inelastic in the short run, and more elastic in the long run.

	Short Run	Long Run
Gasoline	-0.2	-0.5
HH Electricity	-0.1	-1.9
Air Travel	-0.1	-2.4
Intercity bus travel	-2.0	-2.2

Elasticity example:

Washington DC needed money in 1980, and they increased the excise tax on gasoline sold in the district by 6%. If we use the short run elasticity example here, what percent reduction in demand should we predict?

-.2= (% change in Q)/6%, a 1.2% reduction.
In the long run, a 3% reduction.

What happened? Well, six months after implementing the policy, sales of gasoline in the district had reduced 33%. What is the implied price elasticity of demand?
-33%/6%, or -5.5 .

They repealed the tax after five months. What went wrong?